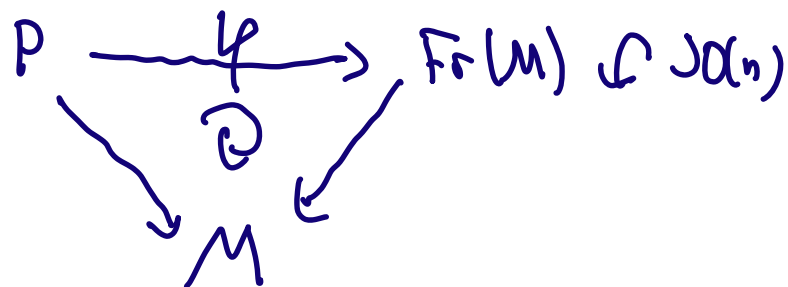


A new approach to noncommutative Riemannian spin geometry

I The classical picture

- Let M be a compact oriented Riemannian manifold of dim n
 - Let TM be the tangent bundle
 - there is a canonical principal $GL(n)$ -bundle associated to TM , the so-called Frame bundle over M
 - orientability + Riem. metric \Rightarrow $Fr(M)$ can be reduced to a $SO(n)$ -bundle
 - $Fr(M) \times_{SO(n)} \mathbb{R}^n \cong TM$
 - $Spin(n)$: = universal covering of $SO(n)$
- $$1 \rightarrow \mathbb{Z}_2 \rightarrow Spin(n) \rightarrow SO(n) \rightarrow 1$$

- Def A spin structure on M is a principal $\text{spin}(n)$ -bundle P over M together with a 2-fold covering $\varphi: P \rightarrow \text{Fr}(M)$ s.t.



- Let $\pi: \text{spin}(n) \rightarrow \text{GL}(\Delta_n)$ be the so-called spin-repr. on the space Δ_n of n -spinors
- $S := P \times_{\text{spin}(n)} \Delta_n$, the so-called spinor bundle over M
- S is hermitian, equipped with an inner product $\langle \cdot, \cdot \rangle$ $\mathcal{H}_S := \overline{\Gamma(S)}^{\langle \cdot, \cdot \rangle}$

Smooth-sections

- $C^\infty(M)$ admits a faithful $*$ -repr. on \mathcal{H}_S by pointwise multiplication
- There is an unbounded s.o. operator D on \mathcal{H}_S with compact resolvent and bounded commutators $[D, f]$, $f \in C^\infty(M)$
- $(C^\infty(M), \mathcal{H}_S, D)$ is called a spectral triple
spectral triple \swarrow Dirac operator

II) The NC picture

- Let B be a unital C^* -algebra
- By a spectral triple on B we mean a $\mathcal{D}_B := (B_0, \mathcal{H}, D)$ s.t.

- 1) B_0 is a dense unital \ast -sub. alg of B
- 2) \mathcal{H} is a Hilbert space carrying a faithful \ast -repr. of B
- 3) D is an unbounded s.o. operator with compact resolvent and bounded commutators $[D, b]$, $b \in B_0$

THM (Connes OP)

(B_0, \mathcal{H}, D) commutative spectral triple
+ extra condition) \Rightarrow

$$(B_0, \mathcal{H}, D) \cong (C^\infty(M), \mathcal{H}_S, D)$$

$$\cdot \Omega^1(P_B) = \left\{ x [D, y] : x, y \in B_0 \right\} \\ \subseteq B(\mathcal{H})$$

Connes' differential 1-form

\cong tangent space

III Research in progress

- Let $D_B = (B, \mathcal{D}, D)$ be a spectral triple
- Let n be the metric dimension
Hilbert B -bimodule

Q1) (Framing)

Is there systematic construction of a NC frame
NCPB $(A, SO(n), \mathcal{D})$ with $A^{SO(n)} = B$
such that $\Omega^1(D_A)^{SO(n)} \cong \Omega^1(D_B)$

Q2) (Spin)

When is there a 2-fold covering
 $(C, \text{Spin}(n), \gamma)$ of $(A, \text{SO}(n), \alpha)$ s.t.

a suitable induction recovers \mathcal{H}
in the spectral triple.

THANK YOU!